Analysis of Gamakams of Carnatic Music using the Computer

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Indian classical music is characterised by movement between and around notes. In particular the Carnatic music system uses a variety of such movements with the collective name *'gamakam'*, each gamakam having its own name. The term 'Gamakam' is defined in *Sangeetha Rathnakara* as a pleasant shaking of a note (*Sangeetha Rathnakara* 3,87). In the present day Carnatic music, ragams can be distinguished by the gamakams even when the basic notes are same. In his work *Sangeetha Sampradaya* Pradarsini (c. 1900), Subbarama Dheekshithar has described in detail the different gamakams (some of which on the basis of techniques used in Veena-playing), assigned symbols for them and used the symbols in the notations for the lyrics given in the book.

Past work on frequency measurements of Indian Music.

Use of electronic or mechanical gadgets to analyse the frequencies used in Indian classical music has been attempted earlier (see Intonation in Present-Day North Indian Classical Music, N.A. Jairazbhoy and A.W.Stone, *Journal of the Indian Musicological Society*, Vol 7 No. 1, March 1976 and *The Grammar of South Indian (Karnatic) Music*, C.S.Ayyar, 1951, pp 138-140 which describes the experiments made in this connection and refers Ayyar's papers published in *Current Science*). The study by Jairazbhoy and Stone covered vocal as well as instrumental music while C.S.Ayyar's study was based on his own violin play. Both these used oscillographs to record the music after it was subjected to filteration to remove the higher harmonics. The records show a graph of the sound amplitude against time. An estimate of the frequency is obtained by counting number of periods over a certain time interval. The oscillations were recorded on a photographic plate using suitable techniques along with a reference wave form for the time measurement and the peaks were observed using microscopes. In a paper by C.S.Ayyar the range of gamakams on certain notes were studied and depicted graphically. ("Microtonal Variations in Frequencies in Karnatic Music –II", *Current Science*, Feb.,1953, 22, 39-40). These studies were significant in the sense that an objective analysis of Indian music was attempted using electronic instruments. (The procedure was apparently laborious and expensive as photographic plates were required.) However, no special attention has been paid in the past for the time durations involved in gamakams although it is essential to measure them in order to delineate a particular gamakam correctly.

The study by Jairazbhoy and Stone showed some interesting results: even steady notes were not held at constant frequencies. There was a leading part which often overshot the note: subsequently the note settled at some reasonably steady frequency. From the one example where details are given in the paper (for a vocal music recording) it is seen that the overall variation was between 272 cycles and 264 cycles (a range of 52 cents or about half a semitone) and even within the steady part the variation was between 271 cycles and 266 cycles (a range of 32 cents). For the purpose of frequency estimation the authors took the part where the note was reasonably steady. The study by C.S.Ayyar also showed variations of frequency in the steady note pa on the violin pa string which varied from 609 to 612 i.e. a variation of 8.5 cents. The lower variation in this case is attributable to the fact that it is the open string of an instrument and therefore even the small variation may be due to changes in bowing pressure. The mean of the entire range was taken as the frequency of the note. In the study by Jairazbhoy and Stone the note Suddha Gandhar of rag Yaman Kalyan was found to be held at different frequencies by different artists. The variation was from 382 cents to 429 cents relative to Shadja. In one case a variation from 382 cents to 403 cents was noticed in the playing of sitar by the same artist. (For an explanation of the cents system to refer to relative frequencies see Appendix D)

Personal Computer for analysing frequencies

The advent of the Personal Computer and sound cards which incorporate Analog Digital Converters has made it very easy to record music in a digital form and analyse it. (See Appendix B for a note on digital sound files). The present article describes the attempts made to analyse some gamakams of ragam Mayamalavagowla of Carnatic music using an ordinary PC with a sound card, recording the music from CD's or directly with a microphone to the PC (to eliminate possible distortions due to defects in the tape transport mechanism of a tape recorder). The frequencies were analysed using different techniques for which a computer program was written. The study covered both the frequency aspect and the time durations of the movements.

Pitch, Frequency and Frequency Estimation

Pitch is the musical sensation felt by the listener while frequency is a measurable physical quantity. Although it is generally presumed that pitch and frequency are identical, enough evidence exists that it is not so. (For information on this subject see Appendix C). The pitch perception of notes varying in frequencies as in the case of gamakams adds a new dimension to the subject.. In the *Conclusions* para. of this article a possible connection between frequencies and pitch perception for notes held with gamakam has been suggested.

Many techniques exist for measuring the frequencies of tones including those containing harmonics. These are described in Appendix A which also explains how the results in this study were arrived at.

Methodology used

The music was stored as a PCM .wav file in the computer either by capturing it from an audio CD directly or by recording it using a microphone. A sampling rate of 11025 Hz was used which can faithfully reproduce frequencies upto 5512 Hz which is adequate for the music recorded and analysed. A computer program was specially written for the purpose of the present study. The program can generate graphs of frequencies against time for the fundamental and the harmonics and create log files of frequencies apart from enabling estimation of frequencies by 3 different techniques -Fast Fourier Transform (FFT), Average Magnitude Difference and manual measurement (see Appendix A). In practical analysis, when there is doubt, the values obtained by FFT were checked with the values returned by the manual measurement techniques and the later were taken as more accurate. However, this technique is laborious and is useful only for the fundamental frequency. For drawing graphs of frequencies over a time period only the FFT technique is of practical use due its speed and automatic generation of the graph to depicti the movement of the melody. The graphs and the log files produced using the FFT technique were used to locate the peaks or valleys in the soundscape, which were subjected to manual method of analysis. For estimating the frequency of the tonic (adhara Shadjam) the mean of a range of values in the log file of a steady note was used.

The commercial CD's have accompaniments which could interfere with the analysis. Only parts of the music were chosen for analysis where the accompaniment was minimum in volume and the main melody was prominent. In the microphone recordings there were no accompaniments.

Ragam Mayamalavagowla

The ragam Mayamalavagowla (in which a beginner learns his or her lessons) uses the notes (in Carnatic terminology) Shadjam (C), Suddha Rishabham (D flat), Anthara Gandharam (E), Suddha Madhyamam (F), Panchamam (G), Suddha Dhaivatham (A flat), Kakali Nishadham (B). Gamakam is used in the 4 notes *ri*, *ga*, *da* and *ni* and occasionally in the note *ma*. In general practice, notes *ri* and *da* are oscillated from around the lower notes *sa* and *pa* respectively. In the ascent while singing *ga*, the note *ma* is reached by an upward movement from below (this gamakam is called '*thirupam'* or '*nokku'*) and then *ga* is held with a '*kampitham'* (simple shake). The note *ni* is held much as *ga* but is usually anchored on the upper note *sa*. In the descent the same gamakams are applied but in reverse order. The purpose of the study is to analyse the manner in which these notes are sung by different artists and try to find some common parameters such as the frequencies involved in the movement, the total range of movement, the durations of upward and downward movements, the total duration of a gamakam compared to the total duration of the note itself.

The music selected for analysis was from (i) a commercial audio CD by D.K.Pattammal, with the Muthuswamy Deekshithar krithi *'Srinathadi*' in raga Mayamalavagowla, (referred to here as Voice 1); (ii) notes sa ri ga ma pa da ni sa sung by Vijayalakshmi Rajaram, an accomplished vocalist (Voice 2), (iii) the same notes sung (Voice 3) and (iv) played on the violin by Madurai S.Balasubramanian a leading accompanist (referred to as Violin). Two more voices have also been analysed but for certain notes only. These belong to Sumithra Vasudev (Krithi Vidhulaku from Thyagaraja CD-ROM) here referred to as Voice 2a and Neyveli Santhanagopalan (Krithi "*Mayatheetha*" by Ponnaiah Pillai) referred to as Voice 2b from a commercial audio CD.

When a note which sounds steady to the ear shows variations in frequency in the analysis, the weighted mean has been taken as the basic pitch of the note (as was done by C.S.Ayyar). The figures were cross checked using the other two techniques. No signifcance was attached to the change in amplitude -taking amplitude into account one can be attempt a separate study. It was found that notes with gamakam could also be characterised by the mean fequency of the entire note (including large rests at the lower or upper end) or by the mean freqency in the regions of the up and down movements. In my experience with generating synthetic Carnatic music with a computer (Synthesizing Carnatic Music with a Computer, Sangeeth Natak, No.'s 133-134, 1999, pp 16-24) I had found that the mean does carry a sense of the pitch of the whole note.

The results of the analysis are tabulated in the four tables annexed to this article. Instead of showing the actual frequencies, the relative frequencies with reference to the Shadjam of the artist are shown in cyclic cents (Appendix D) which makes it easy to compare the extent of movements. The parameters shown are (1) the number of oscillations (up and down movements of pitch) in the note, (2) the mean relative frequency of the entire note, (3) the maximum range of movement in the gamakam, (4) the relative frequencies of the maximum (peak) points in the oscillation of the gamakams ri,ga and da and the minimum(valley) point in the oscillation of gamakam of ni, (5) the mean relative frequency of the gamakam parts excluding the steady parts , (6) the percentage of time spent in oscillation, (7) the average duration of upward movement (8) the average duration of downward movement. In the case of ni the note is anchored to the higher note sa and comes down to the lower point quickly and goes up and the minimum is significant (except in the case of the violin sample).

Gamakam on the (Carnatic) Note Suddha Rishabham

The commonly accepted frequency value for (Carnatic) Suddha Rishabham is 16/15 (112 cents) or a comma less 256/243 (90 cents). The most surprising result of the study is that the peak touched in the gamakams of the vocalists are consistently higher than the theoretical figures. In the case of two artists it is almost a major tone, but in the case of the violin the peaks were around 95 cents i.e.,, near the theoretical figure. In the vocal singing of the violinst the peak frequencies were much lower than the other vocalists and only in one case it exceeded theoretical value of 112 cents (r.f 16/15). However, when we look at the mean of the entire note the relative frequencies are well below the semitone values. In one case it was as low as 11 cents mainly because the artist remained at the lower limit (which was close to sa) for considerable periods. The measurement of means of the oscillating parts alone (i.e. from the point where the pitch starts raising till it comes down to the steady or lowest value) showed lower variations among the artists the figure being between 60 and 90 cents, except in the case of Voice1 and Voice2a where the figure is higher.

As the gamakams were anchored at the lower end on sa the range of movement is also roughly equal to the maximum relative frequency of the highest peak. In some cases the lower end of the movement was slightly higher than sa while in others the voice reaches even below sa.

As regards the durations no common point could be noticed except that mostly the duration for upward movement was less than the duration for downward movement. No common point could also be noticed for the percentage of time spent in the gamakam except that the shorter the duration of the note, the larger is the percentage.

From the listener's point of view the gamakams whose peaks were far higher than the theoretical value did not evoke the sense of higher pitch. Instead the vocal rishabhams gave a sense of greater 'ghana' compared to the rishabham of the violinist which remained mostly within the theoretical frequencies and was felt as subdued or ‘soft’.

The figures of 16/15 or 256/243 usually ascribed to these notes do not appear to have any significance when the note is held entirely as a gamakam without prolonging it. Only in the case of the violin play the peaks of the gamakams are close to these values. This may be ascribed to the fact that the production of note is basically through a physical of movement of fingers and while playing gamakam the artist instinctively moves the fingers to the point where he plays the note when held without gamakam.

Gamakam on (Carnatic) Anthara Gandharam

The relative frequency values commonly assigned to this note are 5/4 (386 cents) and sometimes 81/64 (408 cents, a comma higher)

Unlike in the other three notes, except in the case of the violin, the note was not anchored on either end but oscillated uniformly. In the case of violin it is anchored around gandharam and moves in short bursts upward. However, the lower point was below anthara gandharam in the case of one vocal artist in the descent (where the note ended at ga) and in the case of another vocal artist in the ascent. In the case of one vocal artist the peaks (upto even 596 cents) were consistently higher than the suddha madhyamam (r.f. 498 cents) but in the case of the other artists it was close the madhyamam value. As in the case of suddha rishabham the range of oscillation was much more than a semitone in the case of the vocalists while it was only 80 cents in the case of the violin. The voice recording of the violinist showed a tendency for a larger range of oscillation (114 cents) but less than that of regular vocalists. An interesting feature is that in the case of Voice1 the note went down to an r.f as low as 274 cents (below Carnatic sadharana gandharam or Hindustani komal gandhar) but the overall feeling was one of the correct note held with kampitha gamakam presumably because the mean relative frequency of the entire note was 435 cents and the means of the individual oscillations were around 450 cents. The means of individual oscillations showed the greatest consistency – around 450 cents, except in the case of violin in descent where it was around 410 cents and the voice of the violinist in descent (420 cents). Thus (except in these 2 cases) the mean itself was between anthara gandharam and suddha madhyamam generating a feeling of oscillation between these 2 notes although the actual limits of frequencies touched were quite different.

Gamakam on (Carnatic) Suddha Dhaivatham

The commonly ascribed frequencies for dhaivatham are 8/5 (814 cents) and 128/81 (792 cents). The results of the analysis for gamakam on this note are similiar to those on rishabham to which it is a *samvadi swara*. However in the case of Voice1 the note duration was only half of that for ri and there were only one full oscillation and a second oscillation which led to ni without fully coming down to pa. The range of movement was 279 cents if only the first oscillation is considered and 350 cents if the second oscillation is also taken into account. The range was 150 cents (much more than a semitone) for the other vocalist while the violin remained within the semitone range. The peaks reached by the vocalists were well above the theoretical values of 814 or792 cents while that of the violin was within this range, but the voice of the violinst reached above these values. However the means of relative frequencies within the oscillations fall around 770 cents (except in the case of

Voice 1 which was much higher.) These are comparable to the figures for suddha rishabham if we add 702 cents (the r.f. of *pa*). No common point could be observed as regards the duration except in the case of the violin and the vocal tracts of the violinist which showed uniformly that the upward movement time was much less than the downward movement time. The percentage of time spend in the oscillations for the gamakams themselves is around 60, the shorter the note the larger is the percentage.

Gamakam on Kakali Nishadam

This note is usually held anchored on the upper sa and hence more emphasis was given to measuring the minimum points in the oscillations which are shown in the table. (It was, however, found that the violinist played this note in the ascent anchoring at the lower end and touching the upper peaks quickly, while in the descent it was anchored on the upper note). The theoretical values assigned to the note are 15/8 (1088 cents) and 243/128 (1110 cents). Unlike in the cases of ri and da, the movements did not cross the theoretical values significantly except in the case of Voice 1, where it touched 826 cents (this may be partly attributable to the fact that there was a consonant in the lyric at this point and the note was held only for half *aksharam*). Most of the minima fell within the 2 theoretical ranges and the means of each of the oscillations were also around 1150 cents i.e. 50 cents below upper sa (comparable to 50 cents above ga for anthara gandharam). As before no common featue could be extracted about the durations except that in most cases the descent was slower than the ascent.

A short analysis of ri and ga of Saveri

Although this study covers only Mayamalavagowla, a quick analysis of ri and ga of the ragam Saveri (a *janyam* of Mayamalavagowla omitting ga and da in the ascent) was made. The voice of R.K.Srikantan singing the Thyagaraja krithi ‘Ramabana’ was taken from a CD-ROM on Thyagaraja.

The Rishabham of Saveri is generally said to be very low in pitch and almost at sa. In one sample of ri in Madhya Sthayi which was analysed, there were 2 oscillations, the peaks of which touched 131 cents and 181 cents while the mean frequency of the oscillations were 80 cents and 93 cents which are less than those of Mayamalavagowla for the Voice1 and Voice2a examples but not significantly less than the others (actually more than that of violin). In the same song gamakam on ri in Thara Sthayi with 3 oscillations showed peaks at 125 cents, 60 cents, and 106 cents (pitches reduced to middle octave) and the means of the 3 oscillations were 67 cents, 54 cents and 64 cents. Though the peaks showed a tendency for a lower pitch than the results for Mayamalavagowla, the means are lower than the means of only Voice1 and Voice2a and comparable with others.

The note ga of Saveri is held with extensive gamakam and somewhat lower in pitch than that of Mayamalavagowla. The oscillation of this note at the start of the krithi was from 530 cents coming down to 322 cents and then rising to 464 cents. The bottom point of the oscillation i.e. 322 cents was considerably lower than that of Anthara Gandharam (r.f.5/4 or 386 cents) while the upper end was near Suddha Madhyamam (498 cents). This may be compared with the gamakam on ga in

Mayamalavagowla where the lower end of the oscillation was near 380 cents or higher in most cases (except Voice 1). The range of oscillation i.e. 208 is also higher than the range in Mayamalavagowla (except for Voice 1). It is thus seen that ga of Saveri is oscillated over a wider range and also held at lower position than that of Mayamalavagowla. The lower pitch is confirmed by the fact that the mean of the entire gamakam is only 378 cents in Saveri ga, while it was around 440 cents in the case of Mayamalavagowla. In fact some works ascribe Sadharana Gandharam for this ragam. No doubt more samples require to be analysed especially the gamakams in the phrase 'ri ga ri'.

Conclusions

The study showed that where a note is held by a vocalist with extensive gamakam and where the music does not linger at one of the ends for any significant time, the actual relative frequencies at that end varied widely from artist to artist (and even for the same artist in different places) and the values were also higher (for ri and da) or lower (for ni) than the values usually ascribed for the notes. The ranges of movements of the gamakam were also more than a semitone and varied from artist to artist. Greater consistency was noticed in the mean relative frequencies for the gamakam parts alone. These were quite close to each other and about 70 cents above the lower note.

This surprising result (that the voice reaches much beyond the theoretical frequencies) has also been confirmed by my analysis of more samples of (Carnatic) suddha rishabham and suddha dhaivatham sung by other artists (but not documented in this article).

Where the music stayed for considerable period at one end of the gamakam the relative frequencies of the lower end were close to the frequencies of the lower notes sa and pa respectively in the case of ri and da and the higher note sa in the case of ni. (The note ga however, did not anchor at either end in most cases).

Although the end points of the gamakams were higher or lower than the theoretical values, the listener does not feel so. One possible explanation is that the music went beyond the theoretical values only for periods of less than 60 milliseconds. Some empirical studies by me have shown that there is a threshold of duration for proper perception of pitch of rapid succession of separate notes. While this can vary from person to person the threshold appears to be around 120 milliseconds. Pitch movements in a continuous tone are perceptible for lower durations. This also requires further study and the results can vary from person to person and also the system of music in which the listener is trained.

The wide variations found in the ranges and peaks of the gamakams indicate that there is considerable flexibility in rendering them and these perhaps also imply different styles or schools. *The greater uniformity found in the mean values indicates that the overall perception of a note held with gamakam is perhaps based on the mean value of the frequencies in the oscillating part.* This adds one more dimension to the general study of pitch – frequency relationship Some work has been done on the perception of Vibrato but Vibrato and Gamakam used in Carnatic music are quite different, the former is faster, goes over a lesser range, and consists of more or less uniformly spaced up and down movements of the pitch. The Vibrato is designed to convey only sense of the pitch of a steady note while the Gamakam is specifically meant to convey an oscillation in the pitch and the timings of movements vary widely. The studies on Vibrato seem to indicate that the central frequency conveys a sense of the pitch (see Appendix C).

As regards durations, it was found that the total duration of the note (not shown in the tables) had an influence on the number of oscillations, especially when the notes were sung as solfa and not as part of a lyric. Notes of duration of about 1 second had 2 oscillations, 1.5 seconds had 3 oscillations, 2 seconds had 4 oscillations and so on. This factor is well known and when practicing lessons in higher tempos, the standard method is not to fully speed up the gamakams, but reduce the number of oscillations.

As regards the durations of up and down movements a general trend is that upward movement is faster than downward movement. There were many exceptions to this. The up-down movement durations varied from 50 ms to 200 ms depending on the tempo of the music itself.

A very significant finding is that the violin stuck close to the theoretical frequencies for the upper limits of the gamakam and interestingly the violinist's voice also showed lesser range (but still above the theoretical values). The explanations could be that the production of the note is based on physical movement of fingers and instinctively the artist moves only up to the point where he would be playing the note if held steady.

Future studies

Future studies could cover more complex gamakams. They could also analyse the very same lyric sung by different artists belonging to different schools and also played on different instruments. The constraint placed on the gamakam by a lyric set to rhythm as opposed to the free singing in an alapana or simply singing swarams could also be studied. Studies are also required in musical perception in general for Indian music and Carnatic music, which is phrase oriented, in particular.

Explanation of the tables

In the 4 tables annexed all relative frequencies (r.f.) are in cyclic cents (abbreviated as c) relative to the Shadjam (sa). (See Appendix D for an explanation of the cents system). All durations are in milliseconds (abbreviated as ms). The letters A for ascent and D for descent are used to indicate whether the analysis was made for the particular note in the ascent or the descent; it was not possible to get a suitable point in the music for analysing both ascent and descent in some cases. The mean frequencies have been calculated from the log file created by the program using FFT at predetermined intervals (varying between 10 to 20 milliseconds depending upon the range chosen for analysis) and the mean of the frequencies was calculated using a spreadsheet program. The means of the gamakam parts were similarly calculated from the log file. For actual r.f. values such as the maximum of a peak or the range of oscillation the values of the log file have been further checked using the average magnitude difference and manual measurement techniques.

Table 1. Gamakam on Suddha Rishabham (Hindustani Komal Rishabh - D flat).

Artiste	Number of oscil-lations	Mean R.F.of whole note	Maximum range of move-ment (R.F.)	Maximum R.F. of each oscillation	Mean R.F. within the oscillation	%age of time in the gamakam	Average time of upward move-ment	Average time of down-ward movement
Voice 1	A 2	64 c	228 c	178 c 228 c	87 c 114 c	36%	78 ms	94 ms
Voice 2	A 2 D 3	11 c 44 c	157 c 165 c	122 c 148 c 155 c 155 c 155 c	53 c 62 c 71 c 88 c 97 c	29% 24%	87 ms 59 ms	110 ms 50 ms
Voice 2a	A 2 D 2	76 c 79 c	322 c 198 c	217 c 286 c 136 c 54 c	100 c 127 c 61 c 69 c	67% 30%	75 ms 66 ms	150 ms 54 ms
Voice 2b	A2	39 c	193 c	159 c 199 c	79 c 117 c	59 %	100 ms	124 ms
Violin	A 6 D 5	32 c 59 c	95 c 80 c	85 c 90 c 95 c 95 c 95 c 95 c 95 c 90 c 90 c 85 c 85 c	32 c 32 c 54 c 38 c 43 c 54 c 59 c 48 c 54 c 59 c 54 c	54 % 70 %	116 ms 163 ms	245 ms 262 ms
Voice3 (Violinist's voice)	A 4 D 4	13 c * 47 c *	193 c 174 c	90 c 103 c 77c 152 c 115 c 133 c 103 c 90 c	71 c 22 c 47 c 36 c 39 c 70 c 51 c 50 c	64% 100%	148 ms 100 ms	220 ms 140 ms

* Voice went below sa in the oscillations

For reference – Suddha Rishabham - commonly attributed relative frequencies are 16/15 (112 cents) and 256/243 (90 cents)

Gamakam on Anthara Gandharam (Hindustani Suddha Gandhar - E)

Artiste	**Number of oscil-lations**	**Mean R.F.of whole note**	**Maximum range of movement**	**Maximum R.F. of each upward oscillation**	**Mean R.F. within the oscillation**	**%age of time in the gamakam**	**Average time of upward move-ment**	**Average time of down-ward move-ment**
Voice 1 (descent stopping at ga)	A 3	411 c	232 c	561 c 488 c 596 c	480 c 450 c 502 c	100%	116 ms	127 ms
	D 3	435 c	294 c	532 c 525 c 568 c	435 c 450 c 466 c	80 %	97 ms	121 ms
Voice 2	A 3	441 c	176 c	516 c 496 c 509 c	459 c 431 c 438 c	100%	155 ms	143 ms
	D 3	452 c	134 c	494 c 494 c 500 c	434 c 431 c 441 c	100 %	103 ms	233 ms
Voice 2b	A3	453 c	172 c	538 c 529 c 506 c	462 c 469 c 452 c	94 %	92 c	175 ms
Violin	A 6	406 c	113 c	482 c 439 c 469 c 482 c 494 c 498 c	410 c 423 c 436 c 440 c 444 c 452 c	50%	96 ms	155 ms
	D 5	406 c	80 c	486 c 465 c 452 c 457 c 457 c	411 c 409 c 407 c 408 c 408 c	50 %	80 ms	220 ms
Voice 3	A 4	420 c	114 c	441 c 471 c 471 c 451 c	420 c 420 c 420 c 420 c	100 %	112 ms	197 ms

Voice 1 troughs in ascent 380 c, 364 c: in descent 291c, 274c, 348c
Voice 2 troughs in ascent 339c, 339c, 376c: in descent 366c, 403c, 410c
Violin troughs in ascent 384c, 388c ,410c ,410c 419c, 410c: in descent 384 c, 401 c, 397 c, 397 c, 410c
Voice 3 troughs 378 c, 368 c, 389 c, 357 c
For reference: Anthara Gandharam relative frequencies usually attributed are 5/4 (386 cents) and 81/64 (408 cents).

Gamakam on Suddha Dhaivatham (Hindustani Komal Dhaivath - A flat)

Artiste	**Number of oscil-lations**	**Mean R.F.of whole note**	**Maximum range of movement**	**Maximum R.F. of each upward oscillation**	**Mean R.F. within the oscillation**	**%age of time in the gamakam**	**Average time of upward move-ment**	**Average time of down-ward move-ment**
Voice 1	A 2	826 c$	350 c	984 c 1055 c@	826 c 922 c@	90%	112 ms	74 ms
Voice 2	A 2	730 c	168 c *	855 c 873 c	753 c 770 c	30%	80 ms	82 ms
	D 3	764 c	146 c *	851 c 839 c 834 c	760 c 748 c 771 c	50 %	110 ms	90 ms
Voice 2a	A 2	800 c	285 c	881 c 964 c@	771 c 821 c	100%	88 ms	108 ms
Voice 2b	A2	723 c	242 c	887 c 924 c	780 c 787 c	70 %	58 ms	63 ms
Violin	A 4	723 c	101 c	807 c 787 c 770 c 774 c	756 c 759 c 752 c 756 c	42%	100 ms	230 ms
	D 4	720 c	88 c	794 c 787 c 780 c 780 c	742 c 749 c 737 c 741 c	58 %	118 ms	320 ms
Voice 3	A 4	800 c	135 c	800 c 809 c 841 c 850 c	750 c 758 c 767 c 775 c	62%	115 ms	155 ms
	D 3	792 c	150 c	866 c 850 c 858 c	800 c 783 c 783 c	63%	120 ms	155 c

Voice 1 & 2a @ the second oscillation leads to ni without coming down fully to pa
which may explains the higher values
$ range will be 279 c if the peak of the first oscillation alone is considered
Steady part between the peaks 705 cents (almost panchamam)
Voice 2 - trough between notes in ascent & descent 724 c (22 cents above panchamam)
* range will be 149 c and 127 c if the trough between two oscliiations is taken as the lowest point instead of the steady lead before the peaks.
Violin.- trough in ascent 706 to 709 c: Voice 3 - trough in ascent 715 c
For reference – Dhaivatham relative frequencies usually attributed are 8/5 (814 cents) and 128/81 (792 cents)

Gamakam on Kakali Nishadham (Hindustani Suddha Nishad - B)

Artiste	Number of oscil-lations	Mean R.F.of whole note	Maximum range of movement (R.F.)	Minimum R.F. in each oscillation	Mean R.F. within the oscilla-tion	%age of time in the gamakam	Average time of down-ward move-ment	Average time of upward move-ment
Voice 1	A1*	1047 c	403 c	826 c	992 c	72 %	140 ms	120 ms
	D1**	1175 c	138 c	1082 c	1155 c	77 %	110 ms	60 ms
Voice 2	A3	1155 c	112 c	1092 c 1112 c 1117 c	1131 c 1154 c 1154 c	100 %	118 c	200 ms
	D 3	1161 c	169 c	1083 c 1093 c 1073 c	1163 c 1140 c 1145 c	100 %	115 ms	180 c
Voice 2b	D1	1161 c	250 c	1116 c	1148 c	50%	85 ms	75 ms
Violin@	A 5	1135 c	76 c	1102 c 1125 c 1125 c 1128 c 1125 c	1145 c 1162 c 1149 c 1154 c 1157 c	30 %	110 ms	105 ms
	D 5	1164 c	101 c	1102 c 1128 c 1133 c 1125 c 1128 c	1157 c 1160 c 1161 c 1164 c 1153 c	60 %	168 ms	156 ms
Voice 3	A 4	1187 c 1180 c $	122 c	1112 c 1098 c 1126 c 1126 c	1200 c # 1173 c 1187 c 1180 c	100 %	195 ms	108 ms
	D 5	1160 c		1091 c 1112 c 1098 c 1105 c 1119 c	1160 c 1160 c 1160 c 1160 c 1153 c	70 % (rest leading steady Sa)	168 ms	140 ms

* half note

** quarter note

@ note was anchored on lower end -
maximum values of peaks are 1173 c, 1178 c, 1164 c, 1178 c, 1175c

$ excluding the first oscillation which overshot Thara Shadjam

mean is equal to Thara Shadjam possibly because the maximum overshot Thara Shadjam in the lead up.

For reference: Kakali nishadham relative frequencies usually attributed are 15/8 (1088 cents) and 243/128 (1110 cents)

Explanation of Graphs (Only some samples are included)

Fig .1 shows the frequency graph of the first line of Sri Nathadi covering the 7 notes in the ascending order.

Fig. 2 shows the first line of Sri Nathadi (Voice 1), starting from the middle of note sa, showing whole of ri with 2 oscillations and the beginning of the note ga. 6 harmonics are seen and the oscillations are visible better in the higher harmonics.

Fig. 3 shows the note ga in the ascent (Voice 2) while singing the swarams of Mayamalavagowla. There are 4 oscillations. The vertical resolution of the graph was increased to show greater detail and only the fundamental is shown. The sa frequency was 191.5 cycles per sec. More prominence is found for the upper end of gamakam.

Fig .4. shows the note da played on the violin as a part of the ascending sequence. There are six oscillations and the last three come in quick succession while for the first three is considerable lingering at pa. The upward durations are less than the downward durations.

Fig. 5 shows the note ni sung in the descent of Mayamalavagowla swarams (Voice 2). In this particular case the oscillations are equal without prominence to either end of the gamakam.

Fig. 6 shows the same note as in Fig 4 sung by the violinist. The first 3 oscillations are anchored on the upper end of the gamakam which is upper sa..

Fig. 7 is the graph of the descent of ragam Saveri from Thara Shadjam to Madhya Shadjam. The entire phrase consists mostly of movements only without little prolonging of notes, showing the extent of gamakam used in Carnatic music in the rakthi ragam Saveri.

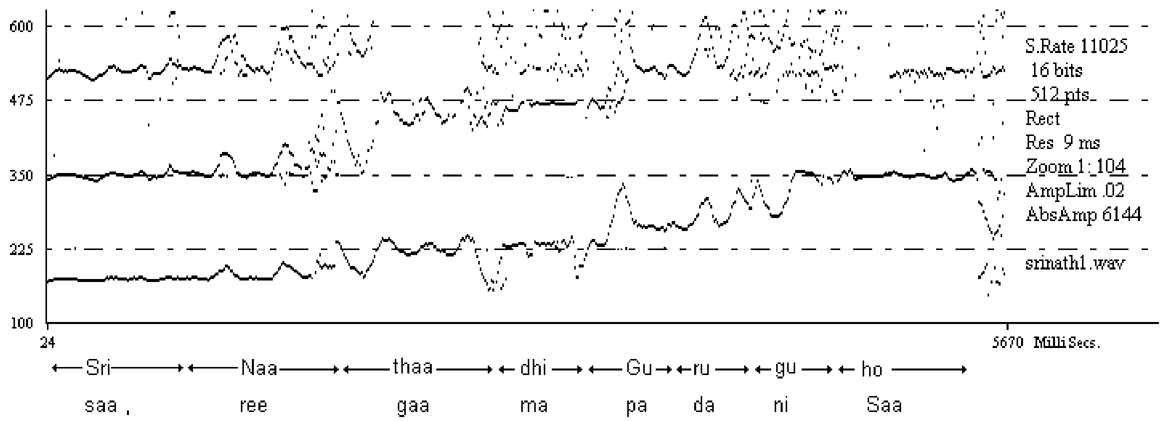

Fig. 1 Part of the first line of Srinaathaadhi Guruguho in Maayamaalavagowla - showing the extensive gamakam.

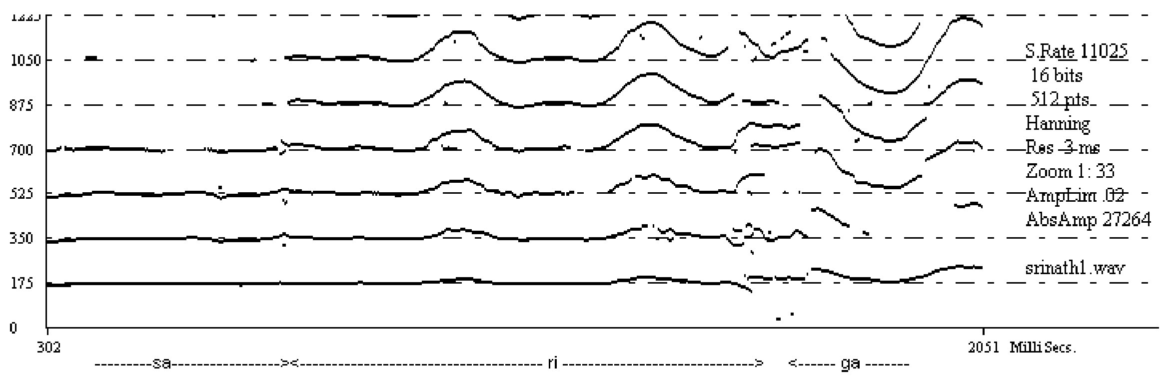

Fig 2. The part 'Sri Naa tha' covering part of sa, the whole of ri and part of ga of the first line of the krithi Sri Nathaadi. Besides fundamental, 5 overtones are shown. The X axis is in millilseconds and the y axis shows the frequency. Tonic sa frequency is 175 hz

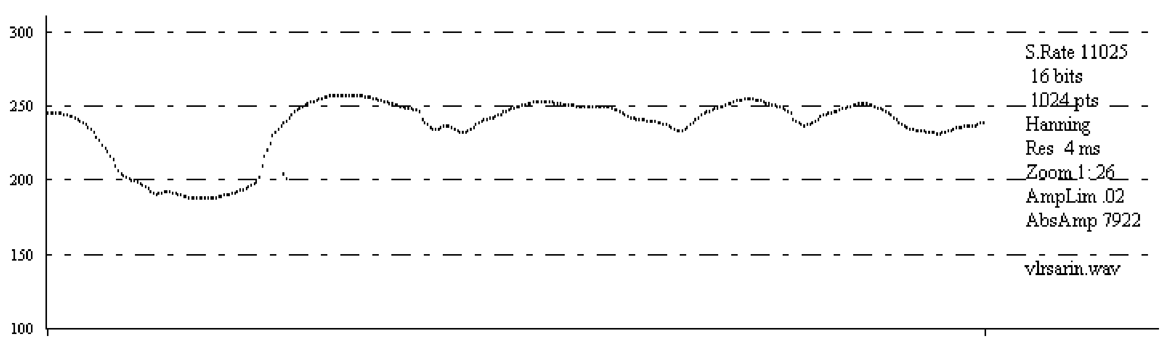

Fig 3. Note ga in the Mayamalavagowla swarams (Voice 2) sung in ascent (sa frequency 191.5)

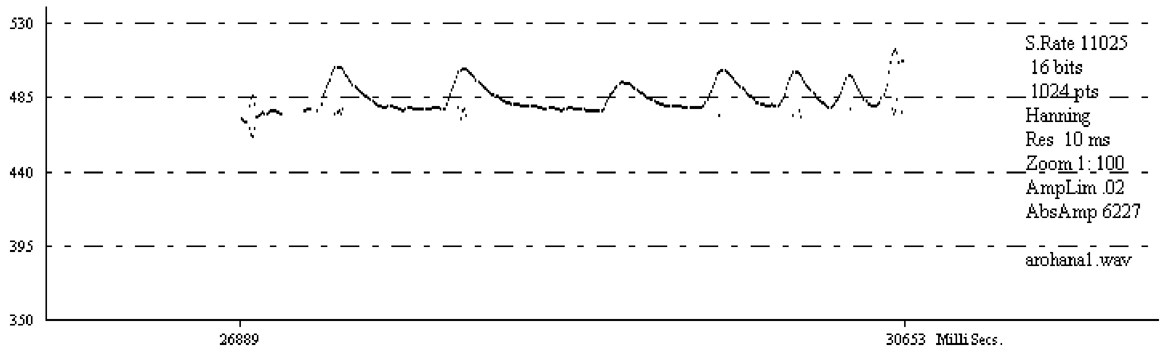

Fig 4. Note da in the Mayamalavagowla swarams played on the violin (ascent). The tonic sa frequency is 318

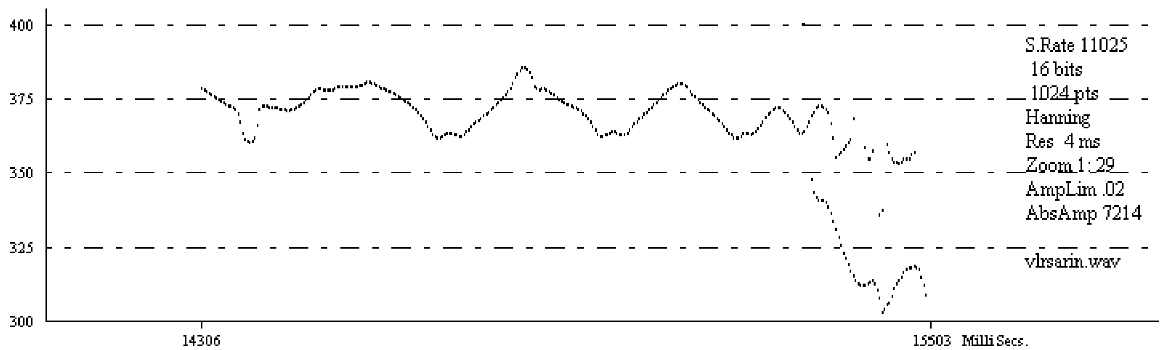

Fig 5. Note ni in the Mayamalavagowla swarams sung in descent (Voice 2) Upper sa freq is 392.4 hz

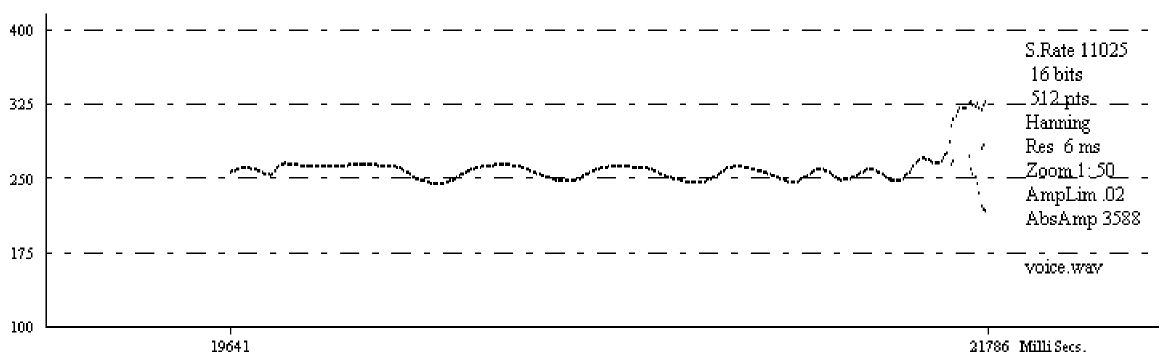

Fig 6. Note ni in the descent of Mayamalavagowla swarams (Voice 3) The upper sa freq was 263 hz

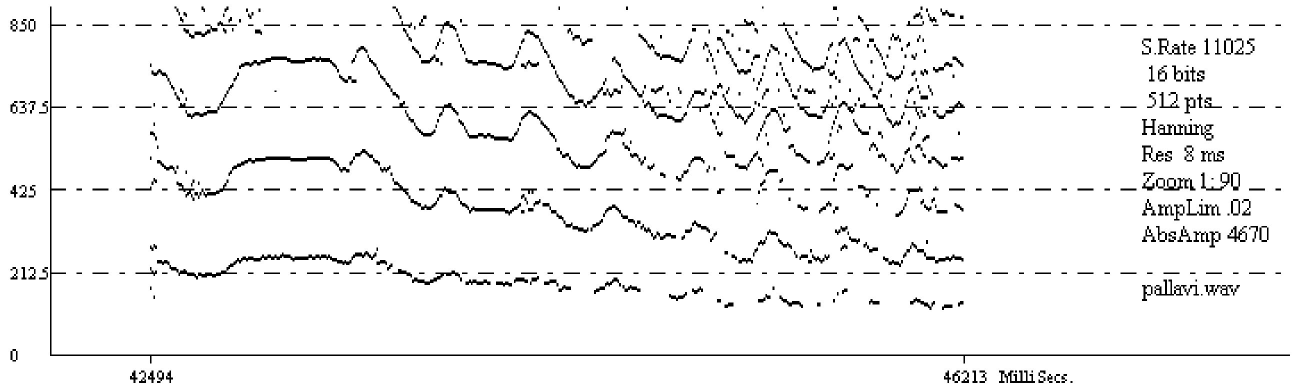

Fig. 7 Descent from Thara Sa to Madhya Sa in Saveri - Krithi Ramabana, Pallavi, Lyric (mana)saa..

Appendix A – Frequency Estimation

A number of techniques are available for estimating the frequency of digitally recorded sound. A brief description of some of the techniques used in this study is given below

The most popular technique is the Discrete Fourier Transform (DFT), which is based on the Fourier theorem that every periodic signal can be represented as a sum of one or more harmonics of the fundamental with suitable weights. The analysis thus shows what frequencies are present in the sound and can show the spectrum (composition of the harmonics of the sound). When dealing with data sampled over a short duration this technique creates certain problems due to sudden cutting off (truncation) of the wave form at the two ends. This is partly taken care off by a process of 'windowing' i.e., modifying the sample so that its amplitudes taper at the ends. Further when the sample analysed does not cover exact number of periods of the sound (which will generally be the case) the frequencies (peaks in the spectrum) do not really correspond to the actual frequencies and certain interpolation techniques are to be used to estimate the frequency.

The accuracy of the estimate improves with increase in the length of sound analysed but when analysing gamakams excessively long durations can miss the fine changes of frequencies. In spite of these drawbacks, very fast computations are possible in the DFT technique by using Fast Fourier Transform (FFT) which requires the number of samples to be a power of 2 such as 128, 256, 512 etc. Because of the high speed of calculations, it is possible to calculate the frequencies at short intervals (such as a few milliseconds) of a sample and use them to plot a graph showing the movement of the melody. This is very useful for a general view of the gamakam and also enables further detailed measurements at the required points.

Autocorrelation is another technique which helps us to a selected number of digital samples with the same number of samples shifted to different points in time. In a perfectly periodic wave there will be 100% match between 2 groups of digital samples separated by one period (reciprocal of the frequency), 2 periods etc. A similar technique requiring less computation is Average Magnitude Difference, which also involves matching the 2 sets of samples by measuring the absolute value of the differences between the amplitudes of the corresponding points of the 2 sets.

A third approach is 'zero crossing'. Here the 2 adjacent points at which the amplitude of the wave crosses zero in a particular direction are identified and the time difference between the 2 points is taken as one period. This can work correctly only if there are no harmonics or the harmonics are week or the original sound is filtered to remove higher frequencies.

In this study the application of this concept was further extended. Periodicity of a wave form is easily recognisable when depicted visually as a graph. The program written (see below) enables blocking certain number of periods (between 2 maxima or minima or 2 points of zero crossing) and calculating the frequency after automatically making corrections for locating the correct maximum, minimum or zero crossing and also interpolating to get a more accurate point (as we are dealing with discrete samples).

The last technique comes nearest to the techniques used earlier by C.S.Ayyar and Jairazbhoy and Stone and can also be taken as the closest to the pitch felt by the listener especially when the fundamental is strong. (It can, however, be argued that the overall pitch sensation is based on all the harmonics and when the overtones are not perfect multiples of the fundamental but quite strong, filtering can distort the results. This problem is discussed later in this appendix)

Although programs for estimating frequencies are available, it was felt that to cater for the special needs of this study a new program had to be written. The program written for this purpose enables the estimation of frequency using all the 3 techniques with improvements. It enables (a) display of the digital information in graphic format (b) change of the resolution of the display so that the wave form can be viewed in required detail (c) selection from the graphic display any part of the data and playing it through the PC sound card (d) subjecting any point to DFT using FFT (Fast Fourier Transform) technique with a choice of number of samples to be covered and subject the DFT peaks to interpolation for estimating the frequency, using different techniques including one developed on empirical basis by me (e) estimating the frequency by Average Magnitude Difference technique at a particular point (f) estimation of the frequency at a particular point by counting peaks, valleys or zero crossings for a few selected periods, with interpolation to improve the estimate (g) display the frequencies(calculated by FFT method) in a block of sound as continuous graph with selectable parameters for resolution, FFT size etc. and save the graph as a picture file and the data as a log file.

The accuracy of the results of using the program was verified by generating sine waves (sound with fundamental and without harmonics) of known frequencies and measuring the frequency with different techniques. For instance a sine wave of 166 Hz gave a frequency of 165.8 (an error of 2 cents only) when subjected to a 512 points FFT (which covers 7.7 seven periods with a duration of 46 milliseconds) and exact value of 166 with 1024 points (92 milliseconds covering 15.4 periods). The A.M.D. technique gave a frequency of 166 (no error) while the technique of measuring time interval between peaks or zero crossings manually gave values of 166.05 and 166.02 respectively - practically no error. More complex synthetic wave forms with overtones also gave comparable results. Synthetic sounds of higher frequencies gave lesser errors as more periods were covered.

A basic problem in measuring frequencies is that different techniques of frequency measurements on digital data can give different results. For instance when we try to find the frequencies in a beating tone obtained by mixing 2 frequencies of (say) 445 Hz and 455 Hz, the commonly used technique of Discrete Fourier Transform (DFT) will show both these frequencies if a sufficiently long duration (in this case 0.4 seconds for a sampling rate of 11 kHz) is taken for the DFT, while the ear will perceive it as a tone of 450 Hz (mean of the 2 frequencies) varying in intensity 10 times a second. If the digital data is projected as a graph and the duration for a selected number of periods is measured, the frequency will be found to be 450 Hz i.e. same as the listener's perception. The technique of Average Magnitude Difference (AMD), would also give the frequency as 450.

It was felt that the frequency estimates generated by DFT and AMD techniques should be further corroborated by manual measurement of certain number of periods and the time taken for the same and arriving at the frequency. In particular an analysis involving different techniques is necessary to confirm unexpected results shown by the DFT technique. The DFT technique is fast if Fast Fourier Transform algorithm is used and for obtaining large number of values at short intervals of time in the music (as required for gamakam analysis) this is the technique of first choice.

Frequencies of overtones: The frequencies of overtones in a given sound may not always be perfect multiples of the fundamental, though in the case of musical sounds they are nearly so. A reasonably accurate measure of the pitch frequency would be to first divide the harmonic frequency by the harmonic number, weight the values by the relative amplitudes of the harmonics and take a geometric mean. While the graphs produced show the harmonics separately, individual frequencies (such as peaks or troughs in the gamakam) were estimated using this technique whenever the higher harmonics were predominant. The results were compared with the frequencies ascertained from other techniques and the most likely figure has been shown in the tables.

Appendix B – A note on digital sound files

Computer files of digital music with extensions such .wav .au etc. (recorded either through the mike attached to the sound card or directly from a CD or a Tape Recorder or even generated synthetically) contain the sound data in the form of numbers representing the instantaneous amplitude of the sound recorded many thousand times a second. The sampling rate decides how many numbers are recorded and stored every second, the higher the number the better the quality. The standard sampling rates are 8000, 11025, 22050 and 44100 times per second. The symbol Hz is used to denote the number of samples per second. The Audio CD recordings are at 44100 Hz. The numbers representing the amplitude usually range from –32768 to +32767, which is referred to as 16 bit recording. If the range is 0 to 255 it is 8 bit recording which can distort the tone in some cases. Separate sets of numbers are required for the 2 channels in the case of stereo recordings.

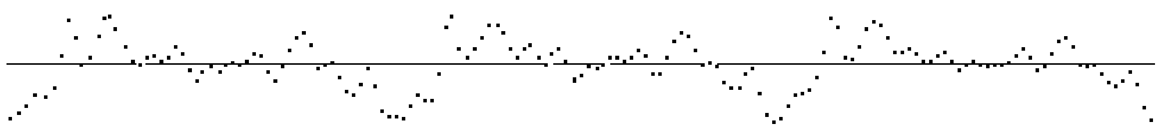

Figure. Digital sound recording shown in graphical form. Each dot represents a sample and the distance of the dot from the central line is the amplitude of the sound at that moment. The sample shown here covers 1/100th of a second and covers about 3 periods implying a fundamental frequency of about 300 cycles per second.

The highest frequency that can be correctly reproduced from a digital recording is half the sampling rate (the Nyquist limit). For faithful reproduction of musical notes the fundamental frequency of the note and the higher harmonics will have to be recorded and reproduced. Thus for a fundamental of (say) 192 cycles per second which is usually the sa of female voices (5 'kattai' of carnatic music), to record up to the 10th harmonic (1920 Hz) the sampling rate has to be at least 3840. For the Thara Panchamam the minimum sampling rate required would be 3 times this. A sampling rate of 11025 can record and reproduce up to 5512 cycles, adequate for carnatic music voices (up to the 9th harmonic of Thaara Panchamam for 5 'kattai'). Radio Broadcasts in the AM band transmit frequencies up to 4500 Hz only. The upper limit of frequency that can be faithfully recorded with ordinary (analogue) tape recorders varies from 5000 Hz to 8000 Hz. Higher tape speed can increase the frequency limit proportionately.

Before recording music digitally, it is necessary to remove from the sound, frequencies above the limit of half the sampling rate, as otherwise they will introduce some new unwanted sounds due the phenomenon called 'aliasing'.

Appendix C. Pitch and Frequency

Pitch is the sensation or the perception of the musical note by the listener while frequency is a physically measurable quantity. Although it is generally presumed that pitch and frequency are identical and the terms used interchangeably, enough evidence exists that sounds of the same frequency may be felt to be in different pitches under certain circumstances.

An increase in intensity of high frequency sounds (over 2000 hz) is perceived as increase in pitch even when the frequency remains constant, while for sounds in lower frequency range the increase in intensity is perceived as decrease of pitch (Rossing, Thomas D., The Science of Sound 2nd Ed, Chapter 7, Addison-Wesley 1990). Terhardt (Terhardt, E., "Calculating Virtual Pitch", Hearing Research 1, 155, 1979), found that a 200 Hz tone was perceived as 20 cents lower when intensity was raised from 60 to 90 decibels while for a 6000 Hz tone the same had the effect of increasing the pitch by 30 cents. A decaying pulsed tone is perceived to be at a higher pitch than a pulse tone of steady amplitude (http://hyperphysics.phy-astr.gsu.edu/hbase/sound/pitch.html.) Harmonic composition also seems to affect the pitch sensation ("Pitch shifts and pitch Deviations", Ernest Terhardt, http://www.mmk.ei.tum.de/persons/ter/top/pshifts.html.) “The perception of the pitch of short pulses differs from that of sustained sounds of the same measured frequency. If a short pulse of a pure tone is decaying in amplitude, it will be perceived to be higher in pitch than an identical pulse which has steady amplitude.”
(http://hyperphysics.phy-astr.gsu.edu/hbase/sound/pitch.html).

Piano notes in higher octaves are ‘stretched’ up to 2% from the theoretical values since due to the stiffness of the strings the higher harmonics are not perfect multiples of the fundamental. (http://www.mmk.ei.tum.de/persons/ter/top/scalestretch.html) and such stretched tuning is found to be more acceptable than theoretically correct tuning.

These were studies on steady notes. Some studies have been made on perception of pitch of notes having vibrato (steady up and down variations in pitch around the basic note) and the results appear to indicate that the perceived pitch is the geometric mean of the frequencies. However, vibrato is also described as as "a rapid alteration of correct and flattened pitch" or “a correct vibrato go(es) from pitch to below and backup"
(http://gigue.peabody.jhu.edu/~ich/research/icmc98/icmc98.vibrato.abstract.html by Ichiro Fujinaga)

While trying to analyse the musical intonation to ascertain the frequencies using gadgets, it has to be remembered that the musician and the listener are interested in the pitch of the notes and the performer has to be judged on the musical quality of the intonation rather than the measured frequencies.

Appendix D. A note on the cyclic cent system for relative frequencies

In music the relation between any 2 notes (interval) is their ratio and not the actual difference in the number of cycles. This makes it difficult to appreciate intervals correctly. For instance it is difficult to say at a glance whether 16/15 is higher or lower than 256/243 and how much do they differ actually. The cyclic cents system converts the relationship of ratios into one of arithmetic addition or subtraction by using the logarithms.

As the octave is a basic interval and as most systems use 12 notes in an octave the relative frequency ratio of 2 of the octave is equated to the 1200 cents. The interval between 2 adjacent notes (a semitone) then becomes 100 cents in the Equally Tempered Scale.

To calculate the cent value of any relative frequency ratio the following formula is used:

cent value = 1200 X log(rf) / log(2)

where rf is the relative frequency ratio. The cent can be calculated using any scientific calculator which provides for logarithms

Some cent values of common relative frequency ratios are given below:

Relative Frequency	Cent Value
1	0
16/15	111.73
9/8	203.91
6/5	315.64
5/4	386.31
4/3	498.05
3/2	701.96
8/5	813.69
5/3	884.36
16/9	996.09
15/8	1088.27
2	1200

A 1% increase in frequency corresponds to an increase of 17.23 cents

To obtain a note which is at an interval of 3/2 (sa to pa) we have to add 701.96 in the cent system. For moving up by 4/3 (sa to ma interval) we have to add 498.05. Thus, the note with r.f. 9/8 (203.91 cents) when increased by 4/3 will give a note of 203.91 + 498.05 = 701.96 (which is 3/2). The above table shows that the equally tempered fifth (pa) with a cent value of 700 is very close to the natural pa (701.96 cents). However, the natural r.f 5/4 (ga) has a cent value of 386.31 is somewhat lower than the equally tempered third with a cent value of 400.

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